Advances in Mechanics
Volume 9, Issue 3, 2021
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Geometry Teaching in Colombia: an Overview from Emerging Didactics for Elementary School Education

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***Article history**:Received 07 October 2021 Revised 05 November 2021 Accepted 08 November 2021*

Abstract

This is a theoretical review that aims to establish a situational framework of the teaching of mathematics in Colombia, as well as reflections and proposals for the teaching of geometry in Primary Basic Education, within the framework of active methodologies. As conceptual referents, Cooperative Learning, the Chronotopia program and the Van Hielle model are assumed for the development of these pedagogical processes (geometry training); all this, in order to identify innovative teaching proposals in the area. It is concluded that the different investigations coincide in that it is required to socialize the teaching of geometry, understanding it as an indispensable value in the transformation of the teaching practice and the student's learning experience.

Keywords: geometry teaching, cooperative work, chronotopy.

Resumen

El presente es un artículo de revisión teórica, que tiene como objetivo establecer un marco situacional de la enseñanza de las matemáticas en Colombia, así como las reflexiones y propuestas para la enseñanza de la geometría en Educación Básica Primaria, en el marco de las metodologías activas. Como referentes conceptuales se asume el Aprendizaje Cooperativo, el programa Cronotopia y el modelo Van Hielle para el desarrollo de estos procesos pedagógicos (formación en geometría); todo ello, con el fin de identificar propuestas innovadoras de enseñanza en el área. Se concluye que las distintas investigaciones coinciden en que se requiere socializar la enseñanza de la geometría entendiéndola como un valor indispensable en la transformación de la práctica de la enseñanza y de la experiencia de aprendizaje del estudiante.

Palabras clave: enseñanza de la geometría, trabajo cooperativo, cronotopia.

INTRODUCTION

The Political Constitution of Colombia (1994), known as the General Law of Education, or Law 115 of 1994, typifies the basic primary education as a level structured on the basis of a common curriculum, made up of the fundamental areas of knowledge and human activity (art. 19, Law 115 /...). 199). Essentially, it seeks to develop intellectual abilities inherent to the learning at the reading, writing, mathematical operations and their application to reality in the framework of a process permanent capacity building cognitive that allow the operation efficient to at life daily. This expresses the purpose of educating for life.

From this conception, arises from the mathematical area in a relevant way because their learning is one of the most important pillars it allows, as well as focusing on the cognitive, develop important skills that are applied on a daily basis in all environments, such as the reasoning, the thinking, the critical thinking, reasoned argumentation and problem solving. Constitutes therefore, a central axis in the development of the human intellectuality, due to a its immediate application in the work. It is therefore important that the student values it as a way to understand the world around him. In this sense:

> The education of mathematics should answer new global and national demands, such as the related to an education for all, the attention at the diversity and interculturality and the formation of citizens and citizenship with the competences necessary for the exercise of their democratic rights and duties (Ministry of National Education, 2006, p. 47).

The educational process in the area of mathematics is still complex and its didactics still faces great challenges. In the case of the Colombian education to be competent in mathematics means to possess analysis, understanding, application of theories and procedures, organized by the Ministry of National Education (2006) in a five-year curriculum process thoughts: (a) numerical thinking; (b) spatial or geometrical; (c) metrical or (d) random or probabilistic; and (e) variational.

However, in Colombia, the results of the assessment in mathematics raised by the TERCE test (UNESCO, 2015). The results show that the average performance of third grade students in the third grade of primary school is average with a score of 694, since it does not differ statistically from other countries such as Ecuador who obtained 703 points; which is maintained for the students in the sixth grade of primary school, whose average score was of 705, without differing with countries like Ecuador (702) and Brazil (709). In geometric domain, whose contents for the third grade were: location in space, points of reference, displacements and transformations, geometric shapes, and cubes, while for the sixth-grade elementary grades become more complex with polygons, axes of symmetry, angles and their classification (among others), the average percentage remains the same but the score is lower. For example, for this last content belonging to level II performance, the country average was 50% and the percentage of successes for Colombia was 52 and Ecuador 56.

The constant concern for optimize mathematical performance in the educational as that of Rondón (2015), who declares that "the school is not forming for the knowledge, and these errors can be found at are evident from the earliest stages educational (primary), where the student is taught under a traditional model that does not arouse interest and does not require a true knowledge" (p. 62). These voices of experts are calling for the educational institutions to traditional procedures and methods, away from the teaching from a mathematics significant for the academic training and life. In line with this, Cruz (2012) refers to as a considerable part of the Ibero-American countries have claims both from teachers and researchers in the field of the low level of motivation for the mathematics in students, coupled with this to a perspective that emphasizes its little functionality in daily life.

In Colombia, research such as Uribe, Cárdenas and Becerra (2014) search at the educational landscape, specifically at basic primary level to conclude on the prevalence of a teaching model in children's mathematical thinking focused on skills acquisition, arithmetic and numeracy skills and numerical knowledge, handling of algorithms related to the four basic operations, memorization of procedures mechanics for the resolution of problems and calculation of perimeter or area of certain polygons. Specifically, in the area of geometry, the traditional way of teaching is described by the authors in the following terms:

> (...) the teaching process of geometry is limited to teaching of some elements of the Euclidean geometry as for example, the handling of certain concepts related to polygons such as the square, the rectangle, the triangle, the circumference and some solids such as the cone, the prisms, the cylinder and pyramid (Uribe, Cárdenas and Becerra, 2014, p.136).

The researchers claim that the area of geometry and spatial thinking has been dispersed in the processes of and curricula. This short view of geometric knowledge excludes didactic, scientific and historical of the same for which it is made. The spatial sense should be reconsidered intuitive in mathematics.

Specifically, the spatial or geometrical thought serves the whole of the cognitive processes through which are constructed and manipulated by the mental representations of objects of space, the relationships between them, their transformations and various translations or material representations.

Researchers such as Bressan, Bosigic and Crego (2010) coincide in pointing out the relevance of their teaching at the basic primary given that the same is a part of everyday life and is used as a in all branches of mathematics, develops visual and spatial perception and even possesses aesthetic and cultural value. Guerrero (2010) reflects on the relationship between the conception of geometry and the way in which it is taught:

> Many of the limitations that our students say about their understanding of the issues of Geometry are due to the type of teaching that have had. Likewise, the type of teaching depends, to a large extent, on the conceptions of the teacher about Geometry, how it is learned, what it means to know this branch of Mathematics and what it is taught for (p. 2).

The significance of geometry at life-long learning makes it imminent to process approach on how the process is learnt. According to Guerrero (op. cit.), in understanding the reasons for their teaching becomes meaningful in geometry.

In this regard, Gamboa and Ballestero (2009), Van Hielle (1986), and Vasco (2019), have also made contributions on the teaching-learning binomial and the need to refer to models that can raise clearly the geometric reasoning produced in the students, in such a way as to facilitate the didactic structuring processes that is being carried out. In this regard, the author points out that:

> From the point of view of the pedagogy and the didactics of the mathematics, I have seen that it is necessary to start from the students' intuitive models and not the purely symbolic expressions from the usual mathematics, seeking, by of course, the crossroads at which the formally valid reasoning separate from the reasoning with and about imaginable mental models (...) (Vasco, 2019, p.17).

Such an approach involves optimizing the systematization and integration between the methodological and theoretical in the context of structuring an instructional process whose ultimate purpose is the use of a specific didactics of geometry that can contribute to the evolutionary process of children's thinking in order to to achieve the necessary skills for the development of perception space. This, in turn, involves the use of active methodologies that allow breaking with the traditional process and involve the social actors of the context in this educational fact in a way that facilitates the scenarios for situated, contextualized and meaningful learning.

On the meaningful learning of the geometry and its application by the students, Gamboa and Ballestero (2009) analyze:

> Geometric knowledge provides of logical resources to the student that allows to make justifications, tests or validations with higher mathematical rigour, which can be when you want to make this same type of conjecture in other areas of mathematics (p. 116).

Immersed in the developments inherent in the research in the field of didactics of mathematics focused on the spatial geometric thinking, various pedagogical proposals emerged, as methodological and didactic alternatives to the traditional teaching landscape which does not even contemplate the most significant postulates in the area; they have been presented as a way of putting diverse practice in the construction of the notion of space. From the contextualization raised, the purpose of this research is to specify the relevant scenarios for teaching of geometry at the basic primary level, for which a documentary review is made, background information is identified and proposals are presented within the framework in the so-called active geometry, without disassociating from the principles of teaching from the geometry that scientifically proven to reveal the didactic and thought process that involves such an important process. It is added to all this journey the raised by Conde (2018), Sanchez (2018), Rico (2018), Frias (2018), in front of the contribution of the developmentalist model by saying that:

From the analysis of the characteristics of the educational community in general, the teaching and learning processes are projected towards the development of a developmental approach, which represents that plan of action that will contribute to the search for reasons or solutions to specific problems of the educational context.

TEACHING GEOMETRY IN BASIC EDUCATION

The teaching of mathematics at all levels of the education system is offered in a curricular structuring of the five types of thoughts, mentioned above. One of them is the spatial thinking and the geometric system, areas that serve as context for the development of the present article. However, there are limitations in relation to the teaching of the mathematics and one of its dimensions as is geometry.

One of them has to do with Castells (2006) in relation to recognizing the existence of an existing social network that is undeniable and inescapable, as well as the need to conform the United Nations Organization Educational, Scientific and Cultural Organization Science and Culture (UNESCO, 2015a) in relation to education for global citizenship, which highlights the need to prioritize the curriculum in areas such as math, reading, writing and transversal competences of the century XXI. Hence authors such as Peñalva, López and Barrientos (2017), refer to the need for these areas to be able to adjust to a century-old education system XXI in which areas such as mathematics and, consequently, geometry, can be a priority, which calls for a specific training for teachers who provide it.

Considering the above and based on the analysis on the didactics of geometry at present it is important to consider that:

> You can no longer teach geometry as a finished product, suppressing all the construction process of such a knowledge and isolating it from the world or from the other areas of the mathematics. It is necessary that the student takes an active role and demand a bit more from him, rather than being a recipient of information (Gamboa and Ballesteros, 2009, p. 133).

Geometric thinking obeys a progressive advance. Initially, in the initial education, the beginning of initial intuitive forms of thinking, even deductive forms and these correspond to the following levels more advanced schoolchildren. Because of this is of paramount value that the environment of mathematical learning is a space to assess children's experiences in mathematical knowledge (in general) and spatial (in particular), such as key element in their process of development and constitution of thought spatial and geometric, as well as a opening to the symbolic universe of the school mathematics. For this reason, it is essential the development of a geometry of more experimental character that coincides with with a child's notion of space surrounded by geometric elements in the home, the street and the classroom.

Vaca's research (2016) entitled Development of a didactic strategy that facilitates interlearning of geometry in students at the level of Secondary Basic Education in the Unidad Educativa Pujili allows to visualize this problem as part of a continental concern; the same was addressed to the application of strategies for the interlearning of supported geometry in Van Hiele's model in the development of the skills with the criterion of performance of curricular block as well as it also presents methods and curricular adaptations for its applicability in the learning of the geometry. Research like this, although it was developed in Ecuador, specifies that one of the obstacles lies in which students express their lack of understanding due to the:

> (...) type of teaching that have The type of the product is also methodology used by the teacher depends on the notions that he has about what Geometry is, how it is learned, that it means to know this branch of Mathematics but above all the concept of the reasons for teaching (p. 21).

Accordingly, schooling should enhance and consolidate the ability of geometrical thinking because its application highlights the acquisition from values such as rigorousness, the sequentiality and organization, therefore, it is necessary to understand the transcendence of geometric thinking beyond the curriculum content in the mathematics taught in the school, which includes not only the context in which it developed but also the South American reality of which Colombia is a constituent part. The same becomes a structure of thought fundamental for daily life because, in essence, the environment is geometrical and everything is geometry.

At the national level, they have also strategies have been designed based on Cooperative Learning of the mathematics; hence the proposal of Álvarez (2017) entitled *Cooperative Learning as a strategy for strengthening the skills in solving problems with multiplicative structures,* which investigates qualitatively on how the cooperative learning allows the active participation of both teachers and students especially gives students a voice, who can dialogue and manage to be participants in the construction of their own knowledge. About the didactic experience implemented by the author reflect:

> Cooperative learning seeks to teach a range of skills to develop of group work, as well as that you are aware that as soon as you the more you have, the more quality and amount of learning is achieved. Seeks to teach students to regulate themselves and be responsible, developing basic job skills and group problem solving (p. 30).

The study concludes that the influence of Cooperative Learning in the development of the mathematical domains also encourages behaviors interpersonal relationships aimed at achieving of individual and collective goals in the accomplishment of a common task. They add, that, although this innovative proposal does not is a panacea for the problems of the teaching of the mathematics, this represents a proposal valid insofar as it is contextualized to the specific groups obtaining changes favorable in the cognitive and social order.

As well as cooperative work has been a useful approach to teaching, learning and interdisciplinarity has also proved to be relevant to the application of notions and concepts from the area of mathematics. Studies such as Basto's and Triana (2017) called *Propuesta for the strengthening of* *the skills of the spatial and geometric thinking a through Arts Education in students at grade fifth at at Quebradón Sur del Municipio Institution Algeciras*, consider the Geometry Active to strengthen thinking space y geometric a through at artistic creations. The researchers refer:

> A teacher with foundations didactic, theoretical and committed with its work generates processes of significant teaching. The interdisciplinarity between the art education and geometry is an important didactic proposal because it generates good processes of teaching by the applicability with situations in (Basto and Triana, 2017, p. 66).

The research emerges from a research genesis in which it was carried out a profound process of reflection on the didactics at of mathematics y its need of transform the praxis educational. This proposal involved link concepts and mathematics processes applicable to artistic education, as are the point, the line, parallelism, perpendicularity, symmetry, rotation, enlargement, representation of figures flat, two-imensional and three-dimensional, among others.

The various teaching proposals of Geometry are supported by a vision where it exceeds the limits of mathematical thinking for also project aesthetic values and cultural. On the interrelation of the science with language and symbols, Theran and Oviedo (2017) propose:

> One of the general trends most widespread today consists of emphasize the processes of own thoughts of the mathematics to deepen their complexity from the strengthening of semiotic representations and move on to the second the mere transfer of the contents (p. 42).

In this regard, it should be added that the playful strategies emphasize sensitivity intrinsic to geometry and its artistic uses. Likewise that the strategies of cooperative learning point to a shared knowledge. Bressan, Bosigic and Crego (2010) agree in analyzing the role of the socialization in the construction of the geometric thinking so they add that their teaching must attend to the use of techniques, procedures and strategies that allow the achievement of the objectives proposed from a new perspective.

ACTIVE METHODOLOGIES: COOPERATIVE LEARNING

As opposed to the traditional schemes of teaching in which students are considered passive recipients of information, new forms of information focused on the student as the center of the educational process (Silva, 2017). A of them involves the use of methodologies active approaches to learning that approach learning as a challenge, requiring from the teacher a perspective and innovative action in their process didactic process (Vásquez-Romero, 2012), as well as the use of strategies cooperative and collaborative that break with the orthodox restrictions of educational process (Rascón and Cabello, 2013).

And the fact is that, as Pujolás and Lake, 2014) "the development of the competences required, necessarily, the cooperative structure of activity in the

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classroom" (p. 7), by how much the competences basic and can be strengthened under a cross-cutting individualistic system or structure and competitive. This is because since the decade last decade is understood that the learning and the construction of knowledge, require processes that transcend to know, because it is necessary to learn to know, learn to love and to feel, to learn to live together, to learn to be, among others (López and Matesantz, 2009).

Hence, research such as that of Herrada and Baños (2018) propose that "Among these active methodologies, the following stand out, for its use in different educational contexts, cooperative learning, that manages to optimize the process of teaching-learning including at highly complex subjects, such as the Mathematics" (p. 3). In view of the progress of the so-called active methodologies in the teaching of mathematics, in which the aim is for students to acquire competences for different areas as personal, social and professional it can be deduced that the approach centred in the learner not only requires a integrated curriculum but of an integrated approach cooperative in and out of the classroom. It is worth mentioning its effectiveness in the development of interpersonal relationships in groups and therefore, cooperative learning promotes inclusive education.

The cooperative methodology develops multiple types of intelligences. The spatial intelligence is enhanced by the group work in which they exchange ideas and observations about space; in this interaction generates the exercise of motor skills as a form of expression; or interpersonal intelligence which favours a consolidation of relations transcendent to the academic space. In the mathematics, Herrada and Toilets state:

> The study of Mathematics entails that students must gain an understanding of advanced processes and complex procedures that explain numerous situations in the real world. It is precisely the complexity of these subjects makes it which makes the use of active methodologies result particularly suitable for students acquire skills that would enable them to personal, social and economic professional challenges (2018, p.4)

The cooperative methodology coordinates their contributions with other proposals such as the problem-based learning and problem-based projects, the case study and the related to corporeality and playfulness. Especially in terms of the playful content in learning cooperative, studies such as Vilches and Gil (2011) reveal that the Learning Cooperative constitutes a tool essential for the study of the especially when combining this methodology with games such as puzzles or other puzzles that require the problem solving.

However, while the physical space is not the only source that generates mathematical thinking, its importance should be highlighted, given that the students elaborate the logical-mathematical space a from the actions they perform on concrete objects in real space. At this point, the so-called dynamic geometry considers that the skills of thinking spatial thinking and system geometric must be oriented in such a way that dynamic and active, because the space must be explored experientially and then to be able to represent and recognize it from abstract form, which implies transformations in the practices of teaching geometry.

On the other hand, Theran and Oviedo (2018) propose that the association between the Van Hiele's contributions and dynamic geometry come to be factors that enhance the dimensions procedural and attitudinal at students, since this type of work "...makes collaboration possible, the setting up of a common, symmetrical mediation and the development of citizenship competences " (p. 48). The researchers set out to investigate the effects of strategies from teaching geometry and the use of the Cabri software. They place the emphasis on the role played by the experimentation that students can make about the materialization of the mathematic objects. Hence they conclude:

> From this perspective, the mathematics is, above all, knowledge - a science in which the method clearly predominates on the content. For this reason, they attach great importance to the study of the issues, in good part adjoining psychology cognitive, which refer to the mental processes of problem solving. (p. 43).

So the role played by the mathematical experimentation is presented as a relevant teaching strategy. This is because when you run dynamic geometry programs students obtain the following important advances in knowledge of geometry, in its understanding and practical usefulness, to a greater extent more than traditional methods.

Van Hiele (1986) constructs a pedagogical alternative for teaching geometry, defining the levels of reasoning of students in this area (descriptive component) and proposing didactic paths that make it possible to move from one level to another (instructional component). The pedagogical model arises from their own teaching experiences as he narrates:

> When I started my career as a mathematics teacher, I soon realized that it was a difficult profession. There were parts of the subject that I was able to explain and even then the students did not understand. (...). In the years following I changed my explanation often, but the difficulties remained. It seemed that I was speaking in a different language. And considering this idea I discovered the solution, the different levels of thinking (p. 39).

The author points out that reaching a level superior thinking makes possible a new order of thinking applicable to new objects or situations. In his theory, he stresses that the transition from one level to another is to a greater degree attentive to the education received at the age or stage of the student. This is how he puts it: "(...) the transition from one level to the next is not a natural process, takes place under the influence of a teaching programme learning. Transition is not possible without learning a new language" (1986: p. 50). For this reason, it is important to organize the process, teaching-learning methods, as well as the designed activities and materials used.

Hence the need for the use of conscious of creative strategies and cooperatives that facilitate the student's acquisition of skills and specific perception and perception skills, Interaction with the space and its geometrical peculiarities, which is why the work with the other becomes essential as it enriches and enhances this type of processes, providing dynamic and interactive perspectives.

Cooperative learning is an essential tool for the study of mathematics and in particular the geometry, because it favors the meaningful learning and scientific culture, improving the classroom climate by

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involving teachers and pupils in a common task. In this regard, Vilches and Gil-Perez (2011) reflect:

> But working in groups not only favors significantly the meaningful learning and the immersion in scientific culture, but also contributes to a good climate of the classroom with the integration of the students and the teacher in a common task, constituting a key instrument for overcoming difficulties and establishing positive cooperative relationships (p. 79).

They highlight the role played by the metacognitive strategies used by students to improve in a way by following cooperative strategies.

THE CHRONOTOPIA PROGRAM

Specifically in Colombia, a program was built called Chronotopia, which represents a proposal for the teaching of the geometry in that country; it is developed by Dr. Carlos Vasco (Vasco, 2006; 2011, 2013, 2019). The author flexes traditional schemes established teaching methods conventionally for education primary, characterized by limiting the geometry to the mere recognition of figures. He claims that thanks to the research in the area of geometry it was determined that the difficulties presented by the students of careers such as physics, engineering and architecture were due not to possessing knowledge in geometry from the primary education. On this he expresses: "but if we want to create mathematics new ones, we can't wait to start to do so at the postgraduate level, but rather we have to start as children and to become children again in order to initiate and conclude these research mathematics cycles" (Vasco, 2011, p. 90). Hence its proposal entitled Chronotopia Program implemented from the application of a discipline in the factual and formal treatment of space time. This discipline, Chronotopy, would include the mathematical disciplines such as topology, topometry, chronology and timing.

This program is based on the corporeal character of thought chrono-spatial. Recognizes the relationship between corporeality and the perception of being integrated into a space and a time and the perception of the mathematic world. The author explains the essence of his project:

> In a new program for the future of what we usually call "geometry', the first proposal is that the beginning of the work privileged for invention and mathematical reflexion should be always that corporealized world of the temporo-spatial intuition, to complete the picture of the visualization with gestures and corporealization (2011, p.90)

He attributes to this that mobility is processed in two ways: in and inter-recreational aspects, in such a way as to that we can orient and perceive speed. Based on this, he places the future of geometry in the childhood because it is a through Corporealized chrono - spatial imagination, made of muscles and bones, to explore the universe of sciences.

This methodology is characterized by Vasco (2017), as an active discipline. Seymour Papert had his encounter with syntonic geometry, which is part of the body,

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which makes it inspired by the need to start from the body and object movement like the real beginning of geometry. Papert (1997), mathematician, educator and prominent scientist, proposed:

> The knowledge that was accessible only through formal processes, is possible to address now at concrete form. And the real magic arises from the fact that this knowledge includes those elements that one need to become a formal thinker (p. 36).

Papert proposed, from constructionism, the use of digital technologies in education, as well as the idea that the child builds his knowledge. In this way, several theory proposals emerged in education based on both the construction of materials and educational resources. The author argues that the active geometric systems are best represented externally with movements and gestures, what is known as syntonic geometry.

In this perspective, the teaching of the geometry in the primary stage would be immersed in a syntonic chronotopy. This children's corporeality and the corporeality of girls in a form of representation of the time-space thinking, translated in programmes of strategies aimed at the visualization and the cultivation of temporo-spatial imagination, containing chronos and topological. The general purpose for which is aspired with this program is raised by the author in the following terms:

> In a new program for the future of what we usually call "geometry', the first proposal is that the beginning of the work privileged for invention and mathematical reflection should be always that corporealized world of the temporo-spatial intuition, which is the that completes the picture of the visualization with gestures and corporealization. In this visualization-corporalization will locates (and quantifies) the spark of the conjecture, the crucible of the conceptualization, the artifice graphic or gestural for the expression and discussion among colleagues, as well as the aesthetic satisfaction of the problem solved (Vasco, 2013, p. 85).

In this sense, the systems of representation in the teaching of geometry immersed in a syntonic Chronotopia with the corporeality of the children and girls would give way to free exploration and guided that leads to the formulation of conjecture, invention, and reinvention of chronotopic concepts and relationships. The geometric systems assets are represented in a better way through movements and gestures. Vasco Uribe (op. cit.) takes as a starting point the mode and mental models applied by children to infer, understand and link concepts; exposes the importance of the perceptions of the central neural system of a chronotopic thinking and initiate what the author calls the mathematical journey that coincides with a model with a theory and that this match to scientific conjecture.

At its proposal is incorporated, also the playful dimension as to create bridges between the emotional and the cognitive and in this way carve out a fruitful way in the teaching of some of the most mathematics alive and responsive to the world and to scientific thought. This has been a constant request from the author in all the scenarios where he has contributed as advisor in education in Colombia. Thus, he has expressed it when he reflects on the challenges of education in this field establishing as one of them the:

> Reconciling the need for high levels from education of mathematics, natural sciences and technologies with the growing young people's apathy towards these areas; with the shortage of qualified teachers for them; with the decrease in the hours and requirements by associations of fathers and mothers (Vasco, 2017, p. 3).

The Chronotopia program would proceed similarly in both primary and secondary school, high school, middle school and university. The chronotopic visualization and corporealization would remain as the beginning of the exploratory experience, with its related production processes of conjecture, argumentation and check to finally return to visualization and corporealization of the solutions. This process would be distinguished in each stage according to the deepening of the arguments, of the conceptualization and formal rigor of expression. For Vasco (2019), it is fascinating to see the way in which the children associate and verbalize mathematical statements from their imaginative association, thus inviting to reflection on the epistemology of mathematics from a child's point of view.

In this order of ideas, the didactic proposals arise inspired by new ways of building the children's geometrical thinking from its temporo-spatial character. Among them is that of Uribe, Cárdenas and Becerra (2014), who constitutes a design and implementation of a set of didactic units that integrate elements of geometry, the spatial thinking and artistic expression:

> Therefore, in the environment of mathematical learning, it turns out fundamental to recognize and value, among other fields, the existence of experiences of the past and the present. Children's spatial awareness as a fundamental element for the initiation of their development and constitution of the spatial and geometric thinking - essential components of the mathematical thinking - and as opening to the symbolic universe of school mathematics. (p.140)

It is a proposal of an interdisciplinary approach that seeks to integrate the new curriculum new ways of knowing mathematics and art, framed within the active methodology. With all this, there is a commitment to integrating a new curriculum, with a innovative know-how school mathematics and art. Within the framework of the active geometry, the authors designed the didactic proposal of Tessellations for children.The tessellations constitute artistic creations whose elaboration fosters skills such as development of spatial thinking, construction of notions and concepts, integrating elements related to three-dimensional art. So the way to innovate in the didactics of geometry has been long-traveled and there is still a long way to go. Vasco's reflections of a new and more effective situation still in question, such as the geometry didactics. Hence he proposes Active Geometry as a new alternative for the teaching of the spatial and geometric thinking skills.

CONCLUSIONS

The above approaches lead to assert that the teaching of geometry cannot continue to develop on its own with blackboard and pencils in the framework of a society essentially marked by the complexity and diversity. The review related to the active methodology in teaching of mathematics and geometry allowed visualize the different contributions in this field. Gamboa and Ballestero (2009) highlight the metacognitive character of this mathematical knowledge characterizing it as an instrument that "allows the human being to solve problems of various kinds and understand a world that offers a wide range of different geometric shapes, in each of the scenarios that make it up, whether natural or artificial" (p. 114) From this, the different proposals coincide in that socialization of teaching is required for geometry, understanding it as a *sine qua non* value, with the possibility of transforming the practice of teaching and the experience of learning.

Active methodologies place the student and their peers at the center of a dynamic, interrelated and syntonic process. Among them, cooperative learning is performing as one of the methods that optimize the teaching-learning process in complex areas such as mathematics, that rescues the cultural value of the aesthetic of geometry, among others. Thus, geometry and the syntonic geometry of the interdisciplinary approaches achieve recognizing the pedagogical experience as a didactic alternative whose purpose is to think about mathematics school and allows to validate new forms to develop the skills of visual perception and build geometric knowledge.

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